Annals of Nuclear Energy 65 (2014) 433–440

Contents lists available at ScienceDirect

Annals of Nuclear Energy

journal homepage: www.elsevier.com/locate/anucene

A method for on-line reactivity monitoring in nuclear reactors

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ARTICLE INFO

Article history:
Received 14 October 2013
Received in revised form 20 November 2013
Accepted 22 November 2013
Available online 4 January 2014

Keywords:
On-line reactivity monitoring
Point kinetics
Stable period

ABSTRACT

In the present work the problem of the on-line monitoring of the reactivity in a source-free nuclear reactor is considered. The method is based on the classic point kinetic model of reactor physics. A relationship between the instantaneous value of the system stable period and the values of the neutron flux amplitude (or the power), of its derivative and of the integral convolution term determining the instantaneous value of the effective delayed neutron concentration is derived. The reactivity can then be evaluated through the application of the inhour equation, assuming the effective delayed neutron fraction and prompt generation time are known from independent measurements. Since the power related quantities can be assumed to be experimental observables at each instant, the reactivity can be easily reconstructed. The method is tested at first through the interpretation of power histories simulated by the solution of the point kinetic equations; the effect of the time interval between power detections on the accuracy is studied, proving the excellent performance of the procedure. The work includes also a study on the sensitivity of the reactivity forecast to the uncertainty on the values of the effective delayed neutron fraction and prompt generation time. The spatial effects are investigated by applying the method to the interpretation of flux evolution histories generated by a numerical code solving the space–time dependent neutron kinetic equations in the diffusion model. Also in this case the method proves to be quite effective in providing good estimates of the system reactivity, except at very short times after the introduction of a perturbation inducing a spatial transient. At last, the effect of the experimental noise is investigated, proving that the consequences in the accuracy of the reactivity prediction can be mitigated by using an adequate differentiation algorithm.

1. Introduction

The real-time monitoring of the reactivity during the operation of a nuclear reactor is of paramount importance in order to be able to detect any situation that may require a timely intervention of the control system to avoid potentially dangerous power transients. The reactivity ρ constitutes a useful integral parameter that characterizes the distance of the multiplying system from the critical state, associated to a unitary value of the multiplication eigenvalue k. Therefore, in classic nuclear reactor kinetics it is defined as the relative distance of the actual system eigenvalue from the unitary critical value:

$$\rho = \frac{k-1}{k}. \tag{1}$$

The concept of reactivity is introduced in a straightforward manner in the simple point kinetic model (Henry, 1975), being related to a change of the neutron balance operator with respect to the stationary configuration through a perturbation-like formula (Gandini, 1987). Considering these equations, it can be seen that the reactivity constitutes the driving force to establish a transient departing from a critical system in steady-state. The point kinetic equations can be easily solved if the value of the reactivity introduced into the system is assumed to be constant in time. The solution for the total neutron population (amplitude or system power) can be written as a combination of exponentials, whose time constants are the solutions of the "inhour equation" (Henry, 1975). Although the point kinetic equations are very simple and cannot account for spatial and spectral transients, they constitute a powerful tool for the analysis of nuclear multiplying systems and are still widely used to provide useful information for their control and stability analysis.

The reactivity is strictly related to the stable period of the reactor, defined as the inverse of the time constant of the exponential evolution, once the system has reached the final asymptotic portion of the transient following a constant perturbation from the critical state. When such an asymptotic behavior is reached, the system shall evolve according to a single exponential whose time constant is the supremum of the set of solutions of the inhour equation. Such time constant is known as the fundamental solution of the inhour equation. In this condition only, the stable period is simply the logarithmic derivative of the system power. This is not true in the non-asymptotic portion of the transient and, moreover, if the reactivity changes with time.

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http://dx.doi.org/10.1016/j.anucene.2013.11.039

In a nuclear reactor, neutronic measurements give information on the evolution of the neutron population and, hence, of the power generated by the fission process. From the flux signals, either the reactivity or the stable period are deduced, to be able to forecast the power evolution and to provide indications on the control operations necessary to keep the system in safe conditions. The problem of reconstructing the reactivity from flux measurements constitutes a classical inverse problem, very well known in nuclear engineering, and a great deal of literature can be found for both initially critical (Ansari, 1987) and subcritical (Carta and D'Angelo, 1999) systems. Pulsed experiments have shown to be particularly suited for the determination of the subcriticality level of a system (Bell and Glasstone, 1970). However, they are obviously unsuited to provide on-line information throughout the normal operation.

This work presents a method to reconstruct the reactivity in a nuclear reactor in the absence of an external neutron source. The method is based on the mathematical properties of the point kinetic equations, that allow to infer the instantaneous value of the fundamental time constant. Afterwards, through the inhour equation, the reactivity can be readily derived.

The method proposed is particularly suitable for on-line reactivity monitoring, since it is established on neutronic measurements and thus it is direct and passive. No intervention on the system is required and the information on the reactivity is given continuously in time. The reactivity measurement is prompt, since it is not related to any observation of the thermal state of the system. Although the point kinetic integral parameters (effective delayed neutron fraction and mean prompt generation time) are needed to apply the algorithm, the procedure yields absolute values of the reactivity for the case of source-free reactors being considered in this work.

In the following section, the theoretical foundations of the method proposed are presented. The method is then assessed by studying its performance in a point reactor at first. Afterwards, the possibility to reconstruct the reactivity when applied to more realistic spatial transients is discussed. The spatial transients are simulated by a numerical neutron kinetic code (Dulla et al., 2013). The robustness of the method when analyzing measurements affected by experimental uncertainty is also studied. The results prove that the method can be a useful tool for on-line reactivity monitoring in the operation of nuclear reactors.

2. Fundamentals of the method proposed

The method proposed is based on the point kinetic model. In the following, only one family of delayed neutron precursors is assumed for the sake of simplicity, being the extension to more families rather straightforward and considered later. The algorithm developed in the following aims at reconstructing the instantaneous value of the reactivity in an evolving system on the basis of the relationship between the instantaneous values of the state variables (power and delayed neutron precursor concentrations) and the asymptotic situation that would be reached if the reactivity would maintain that same value.

The point kinetic system of ordinary first-order differential equations is written as (Akcasu et al., 1971):

$$\begin{cases} \dot{P}(t) = \dfrac{\rho-\beta}{\Lambda}P(t) + \lambda C(t) \\ \dot{C}(t) = \dfrac{\beta}{\Lambda}P(t) - \lambda C(t), \end{cases} \tag{2}$$

where P is the amplitude of the neutron population (or the system power) and C is the effective delayed neutron precursor concentration. The parameters β, Λ and λ are the effective delayed neutron fraction, the effective prompt neutron generation time and the delayed neutron precursor decay constant, respectively. Dotted quantities denote time derivatives. It is clear from the above equations that, if the system is critical (hence $\rho = 0$), it reaches a steady-state equilibrium condition when $P/C = (\Lambda\lambda)/\beta$. This equilibrium state is usually assumed as an initial condition from which the system is starting a transient following a perturbation that introduces a non-zero reactivity ρ.

The effective precursor concentration can be obtained by time integration of the second equation in system (2):

$$\begin{aligned} C(t) &= C(0)\exp(-\lambda t) + \frac{\beta}{\Lambda}\int_0^t P(t')\exp(-\lambda(t-t'))dt' \\ &= C(0)\exp(-\lambda t) + \frac{\beta}{\Lambda}I(t). \end{aligned} \tag{3}$$

The focus of the procedure presented in the following is to determine the instantaneous value of the reactivity corresponding to a certain state (power and delayed neutron concentration) in the evolution of the system. Supposing a given constant reactivity, it is useful to consider the system of equations written in compact matrix form as:

$$\frac{d|X(t)\rangle}{dt} = \widehat{A}|X(t)\rangle, \tag{4}$$

where the unknown (column) state vector (defined by the power P and the effective delayed neutron concentration C) and the system characteristic matrix are introduced as

$$|X(t)\rangle = \begin{vmatrix} P(t) \\ C(t) \end{vmatrix}; \qquad \widehat{A} = \begin{pmatrix} \dfrac{\rho-\beta}{\Lambda} & \lambda \\ \dfrac{\beta}{\Lambda} & -\lambda \end{pmatrix}. \tag{5}$$

The solution of the problem can be obtained using the eigenvectors of the matrix $\widehat{A}$ to express the unknown state vector (Ravetto, 1997). Therefore, the following eigenvalue problem is preliminary solved:

$$\|\widehat{A} - \omega\widehat{I}\| = 0. \tag{6}$$

The above is the well-known inhour equation

$$\rho = \omega\Lambda + \frac{\omega\beta}{\omega+\lambda} \tag{7}$$

and its two real, distinct solutions can be ordered in such a way that $\omega_1 > -\lambda > \omega_2$. The eigenvalue ω_1 is known as the fundamental eigenvalue, being associated to the most persistent portion of the evolution. Correspondingly, the (column) eigenvectors can be explicitly written and they constitute the most suitable base to express the state vector. However, they are not orthogonal, hence the (row) adjoint vectors are needed to be able to decouple the equations for the components of the solution along each eigenvector; it is rather easy by simple algebra to find out the following expressions for the un-normalized direct and adjoint eigenvectors:

$$|U_k\rangle = \begin{vmatrix} 1 \\ \dfrac{\beta/\Lambda}{\omega_k+\lambda} \end{vmatrix}; \qquad \langle U_h| = \begin{vmatrix} 1 & \dfrac{\lambda}{\omega_h+\lambda} \end{vmatrix}; \qquad h,k = 1,2. \tag{8}$$

The general solution can be written as (Ravetto, 1997):

$$|X(t)\rangle = \sum_{k=1}^{2}\frac{\langle U_k|X(0)\rangle}{\langle U_k|U_k\rangle}\exp(\omega_k t)|U_k\rangle. \tag{9}$$

By taking the first component of the above vector, one can explicitly write down the solution for the system power as a superposition of exponentials. For large values of the time, say t^*, following the introduction of the perturbing reactivity ρ, recalling the ordering of the eigenvalues, the system reaches an asymptotic evolution given by one exponential only, involving the fundamental eigenvalue ω_1, as:

$$P(t^*) \cong \frac{\langle U_1|X(0)\rangle}{\langle U_1|U_1\rangle}\exp(\omega_1 t^*). \tag{10}$$

Obviously, the same asymptotic evolution is also reached if one consider the state at any $t \ll t^*$ as a new initial state; therefore it is possible to write:

$$P(t^*) = \frac{\langle U_1|X(t)\rangle}{\langle U_1|U_1\rangle}\exp(\omega_1(t^* - t)). \tag{11}$$

Comparing Eqs. (10) and (11) and eliminating the $\exp(\omega_1 t^*)$ term, a relationship involving the power and the delayed neutron concentration at all instants can be obtained (Corno et al., 1986):

$$\left[P(t) + \frac{\lambda}{\omega_1 + \lambda}C(t)\right]\exp(-\omega_1 t) = \langle U_1|X(0)\rangle = \text{const}. \tag{12}$$

A further derivation leads to the following relationship:

$$\dot{P}(t) - \omega_1 P(t) + \frac{\lambda}{\omega_1 + \lambda}\left[\dot{C}(t) - \omega_1 C(t)\right] = H(\beta, \Lambda, \omega_1) = 0, \tag{13}$$

which meets the initial objective of the task. It must be noted that Eq. (13) yields two roots; our interest is focused on the leading root indicated by ω_1. Formula (13) constitutes an instantaneous connection between the values of the state variables P and C, which are to be considered as experimental observables, and the value of the fundamental eigenvalue of the system, inverse of the stable period, which is the largest root of the algebraic second-degree equation above. Using the inhour equation (7), the reactivity can then be reconstructed from the value of ω_1. It must be observed that formula (13) involves the quantities P and $\dot{P}$ that can be considered to be directly measured in the system, together with C that can be obtained through an integral functional of the power as given in formula (3) and $\dot{C}$ that can be obtained from the precursor balance equation in the system (2).

On passing, it is interesting to note that a further derivation of Eq. (13) leads to the following property:

$$\begin{vmatrix} C & P \\ \dot{C} & \dot{P} \end{vmatrix}\omega_1^2 + \begin{vmatrix} C & P \\ \ddot{C} & \ddot{P} \end{vmatrix}\omega_1 + \begin{vmatrix} \dot{C} & \dot{P} \\ \ddot{C} & \ddot{P} \end{vmatrix} = 0, \tag{14}$$

which can also be written in the determinant form:

$$\begin{vmatrix} \omega_1^2 & \omega_1 & 1 \\ \ddot{P} & \dot{P} & P \\ \ddot{C} & \dot{C} & C \end{vmatrix} = 0. \tag{15}$$

The above two equations, valid for both eigenvalues and here applied to determine ω_1 only, establish an inverse relationship between the eigenvalues and the values of the components of the state vector, together with their first and second order derivatives. This property of the solution of systems of differential equations has long been known for various applications in mechanics (Krilov, 1931). Of course, if more than one family of delayed neutron precursors is introduced into the model, higher-order derivatives shall be involved in such relationships, making them hardly usable in experimental evaluations.

3. Validation of the method for reactivity monitoring

In order to validate the above formulae, transients have been evaluated for systems characterized by known reactivities, using different reactor kinetic models and solving the equations by a straight analytical approach, whenever possible, or by numerical techniques. By taking the resulting values of the power and its derivative, the effective delayed neutron concentration and its derivative can be reconstructed as shown in the previous section. This operation requires the knowledge of the kinetic parameters of the system, namely the effective delayed neutron fraction and the effective generation time. In practical applications, these quantities are supposed to be known from independent measurements (Pepyolyshev, 2008; Kuramoto et al., 2007). In the present validation procedure these quantities are assumetd to take the same values as the ones adopted in the point kinetic model used to generate the power evolution.

3.1. Reactivity reconstruction for a point reactor

For validation purposes, the signal to be interpreted is generated by the numerical evaluation of the analytical expressions for the solution of the point reactor equations using one family of delayed neutron precursors, as from formula (9). If the exact solution is introduced into Eq. (13), which is based on the point model, the exact fundamental eigenvalue shall be obtained at each instant t, as expected. Of course, this holds only if the reactivity is constant in time. On the other hand, if the reactivity changes with time, the accuracy of the technique is affected by the rapidity of the change which induces a state of disequilibrium between neutrons and precursors. The results presented later will show that, in the case of a step insertion, good results can be obtained even in the early stage of the transient. Table 1 shows how the fundamental eigenvalue may significantly differ from the logarithmic derivative of the power in the first phase of the transient, when contributions from higher-order eigenvectors are still important, and only later, when the dynamic equilibrium between neutrons and precursors is reached, it approaches the expected exact value. At the very beginning of the transient the logarithmic time derivative is dominated by the prompt jump effect (Akcasu et al., 1971).

In real applications the power signal is sampled at certain times and derivatives and integrals should be evaluated numerically. To investigate the role of numerics, derivatives are computed by finite differences and the integral appearing in Eq. (3) by the trapezoidal rule. For different values of the reactivity, some results are presented in Table 2, where ρ_{exp} denotes the "experimental" reactivity reconstructed from the power signal through the method here proposed. In this Table, as in the following ones, all the values for the difference $(\rho_{exp} - \rho)$ smaller than 1.0E−4 are set to zero. As can be seen, the performance of the method is excellent. Excellent results are obtained using even larger values of the time mesh, as is shown in Table 3.

It is worth also to analyze how the values of the effective delayed neutron fraction and prompt generation time affect the performance of the technique. The value of the effective delayed neutron fraction has a little effect and good results can always be obtained. In any case, results improve for small values of β. Good results are also obtained even if the value of the effective prompt generation time is varied on a large range of values, proving the procedure applicable to both thermal and fast systems. Table 4 reports typical results. At the beginning of the transient, the technique yields better results in the case of a longer mean generation time, which implies a less prompt response, while later an opposite situation is encountered, due to a faster decay of the contribution of higher-order eigenvectors in such a case. This conclusion applies for a large range of reactivity insertions.

3.2. Sensitivity to the uncertainties on the values of effective delayed neutron fraction and effective generation time

The relationship between the fundamental eigenvalue and the system reactivity is established by the inhour equation (7). To reconstruct the reactivity, the values of the effective delayed neutron fraction and prompt generation time are needed. As already mentioned, these quantities are supposed to be known from independent measurements. Therefore, it is important to estimate the sensitivity of the reactivity to such parameters.

Table 1
Evolution of the logarithmic derivative of the power and comparison with the fundamental system eigenvalue for different values of the inserted reactivity. The following data are assumed: $\beta = 650$ pcm, $\Lambda = 10^{-6}$ s, $\lambda = 0.1$ s^{-1}; the calculation is fully analytical.

t (s)	$\rho = -100$ pcm $\omega_1 = -1.333E-02$ s^{-1} $\dot{P}/P$ (s^{-1})	$\rho = 100$ pcm $\omega_1 = 1.818E-02$ s^{-1} $\dot{P}/P$ (s^{-1})	$\rho = 300$ pcm $\omega_1 = 8.571E-02$ s^{-1} $\dot{P}/P$ (s^{-1})
1.00$E-05$	$-9.368E+02$	$9.374E+02$	$2.814E+03$
1.00$E-04$	$-5.081E+02$	$5.357E+02$	$1.687E+03$
4.90$E-04$	$-2.915E+01$	$5.776E+01$	$3.171E+02$
1.50$E-03$	$-2.951E-02$	$2.517E-01$	$8.883E+00$
5.00$E-03$	$-1.333E-02$	$1.818E-02$	$8.575E-02$
1.00$E-02$	$-1.333E-02$	$1.818E-02$	$8.571E-02$
1.97$E-02$	$-1.333E-02$	$1.818E-02$	$8.571E-02$
2.00$E-02$	$-1.333E-02$	$1.818E-02$	$8.571E-02$

Table 2
Performance of the method using numerical evaluations of derivatives and integrals with a time mesh Δt of 10 μs. Other data as for cases of Table 1.

ρ (pcm)	exact ω_1 (s^{-1})	t (s)	ρ_{exp} (pcm)	$(\rho_{exp} - \rho)$ (pcm)
−100	$-1.333E-02$	1.0$E-05$	−96.6	$3.4E+00$
		1.0$E-04$	−98.1	$1.9E+00$
		1.0$E-03$	−100.0	$2.3E-03$
		1.5$E-03$	−100.0	0
		1.0$E-02$	−100.0	0
		5.0$E-02$	−100.0	0
		1.0$E-01$	−100.0	0
		1.5$E-01$	−100.0	0
100	$1.818E-02$	1.0$E-05$	97.5	$-2.5E+00$
		1.0$E-04$	98.6	$-1.4E+00$
		1.0$E-03$	100.0	$-9.3E-03$
		1.5$E-03$	100.0	$-6.0E-04$
		1.0$E-02$	100.0	0
		5.0$E-02$	100.0	0
		1.0$E-01$	100.0	0
		1.5$E-01$	100.0	0
300	$8.571E-02$	1.0$E-05$	295.1	$-4.9E+00$
		1.0$E-04$	297.1	$-2.9E+00$
		1.0$E-03$	299.9	$-8.6E-02$
		1.5$E-03$	300.0	$-1.5E-02$
		1.0$E-02$	300.0	0
		5.0$E-02$	300.0	0
		1.0$E-01$	300.0	0
		1.5$E-01$	300.0	0

Table 3
Effect of the time mesh Δt. The same data as for Table 1 are used, with a reactivity insertion of 200 pcm.

t (s)	$\Delta t = 10^{-5}$ s $(\rho_{exp} - \rho)$ (pcm)	$\Delta t = 10^{-4}$ s $(\rho_{exp} - \rho)$ (pcm)	$\Delta t = 10^{-3}$ s $(\rho_{exp} - \rho)$ (pcm)	$\Delta t = 10^{-2}$ s $(\rho_{exp} - \rho)$ (pcm)
1$E-05$	$-4.2E+00$	–	–	–
1$E-04$	$-2.4E+00$	$-2.1E+01$	–	–
1$E-03$	$-3.4E-02$	$-3.0E-01$	$-1.2E+00$	–
1$E-02$	0	0	$5.8E-03$	$9.5E-02$
1$E-01$	0	0	$5.7E-03$	$9.4E-02$

Table 4
Effect of the effective prompt generation time for $\beta = 650$ pcm. The transient is following a reactivity insertion of 150 pcm.

t (s)	$\Lambda = 10^{-4}$ s $(\rho_{exp} - \rho)$ (pcm)	$\Lambda = 10^{-6}$ s $(\rho_{exp} - \rho)$ (pcm)
1.0$E-05$	$-3.75E-02$	$-3.46E+00$
1.0$E-04$	$-3.73E-02$	$-2.00E+00$
1.0$E-03$	$-3.51E-02$	$-1.91E-02$
1.5$E-03$	$-3.40E-02$	$-1.57E-03$
1.0$E-02$	$-2.03E-02$	0
5.0$E-02$	$-2.40E-03$	0
1.0$E-01$	$-1.92E-04$	0

To carry out this study and evaluate the sensitivity coefficients of ρ to β (S_β) and Λ (S_Λ), one can directly refer to Eq. (7) in order to obtain:

$$S_\beta = \left|\frac{\delta\rho}{\delta\beta/\beta}\right| = \left|\frac{\omega_1}{\omega_1+\lambda} + \left[\Lambda + \frac{\beta\lambda}{(\omega_1+\lambda)^2}\right]\frac{\partial\omega_1}{\partial\beta}\right|\beta, \tag{16}$$

and

$$S_\Lambda = \left|\frac{\delta\rho}{\delta\Lambda/\Lambda}\right| = \left|\omega_1 + \left[\Lambda + \frac{\beta\lambda}{(\omega_1+\lambda)^2}\right]\frac{\partial\omega_1}{\partial\Lambda}\right|\Lambda, \tag{17}$$

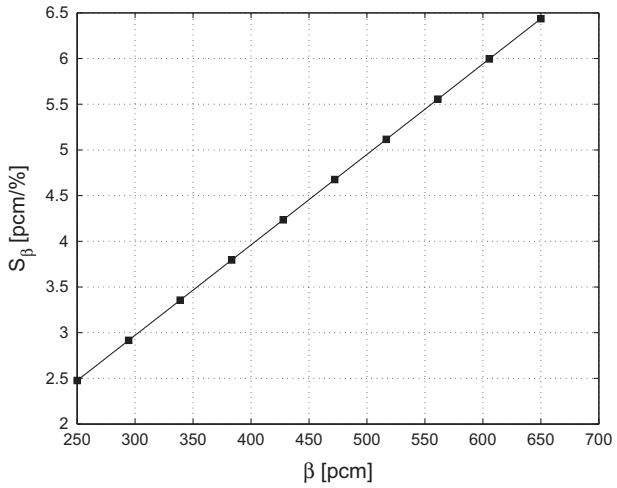

Fig. 1. Sensitivity S_β for $\rho = 200$ pcm and $\Lambda = 10^{-6}$ s at $t = 10^{-1}$ s.

where

$$\frac{\partial\omega_1}{\partial\beta} = -\frac{\dfrac{\partial H(\beta,\Lambda,\omega_1)}{\partial\beta}}{\dfrac{\partial H(\beta,\Lambda,\omega_1)}{\partial\omega_1}}, \quad \frac{\partial\omega_1}{\partial\Lambda} = -\frac{\dfrac{\partial H(\beta,\Lambda,\omega_1)}{\partial\Lambda}}{\dfrac{\partial H(\beta,\Lambda,\omega_1)}{\partial\omega_1}} \tag{18}$$

$$\frac{\partial H(\beta,\Lambda,\omega_1)}{\partial\beta} = \frac{\lambda}{\Lambda}\left(\frac{P}{\lambda+\omega_1} - I(t)\right) \tag{19}$$

$$\frac{\partial H(\beta,\Lambda,\omega_1)}{\partial\Lambda} = \frac{\lambda\beta}{\Lambda^2}\left(-\frac{P}{\lambda+\omega_1} + I(t)\right) \tag{20}$$

$$\frac{\partial H(\beta,\Lambda,\omega_1)}{\partial\omega_1} = -P\left(1 + \frac{\lambda\beta}{\Lambda(\lambda+\omega_1)^2}\right). \tag{21}$$

Some results for S_β are presented in Fig. 1. The sensitivity shows an almost linear behavior with respect to β and it turns out that such behavior does not change significantly with respect to ρ, Λ and with time. The behavior of S_Λ is shown in Fig. 2. It appears that this sensitivity increases for increasing values of β, while it decreases for increasing values of the reactivity. In practical applications, current methods allow to reach relative errors in the measurements of effective delayed neutron fractions and prompt generation times of the order of few percents (Baeten et al., 2001).

3.3. Effect of the number of delayed neutron families

The values of delayed neutron data may play an important role in determining the accuracy of the reactivity estimation (Geslot et al., 2007). In particular, the number of the delayed neutron families constitutes an important issue. The previous analysis has been restricted to the case of only one family. In the present section, the

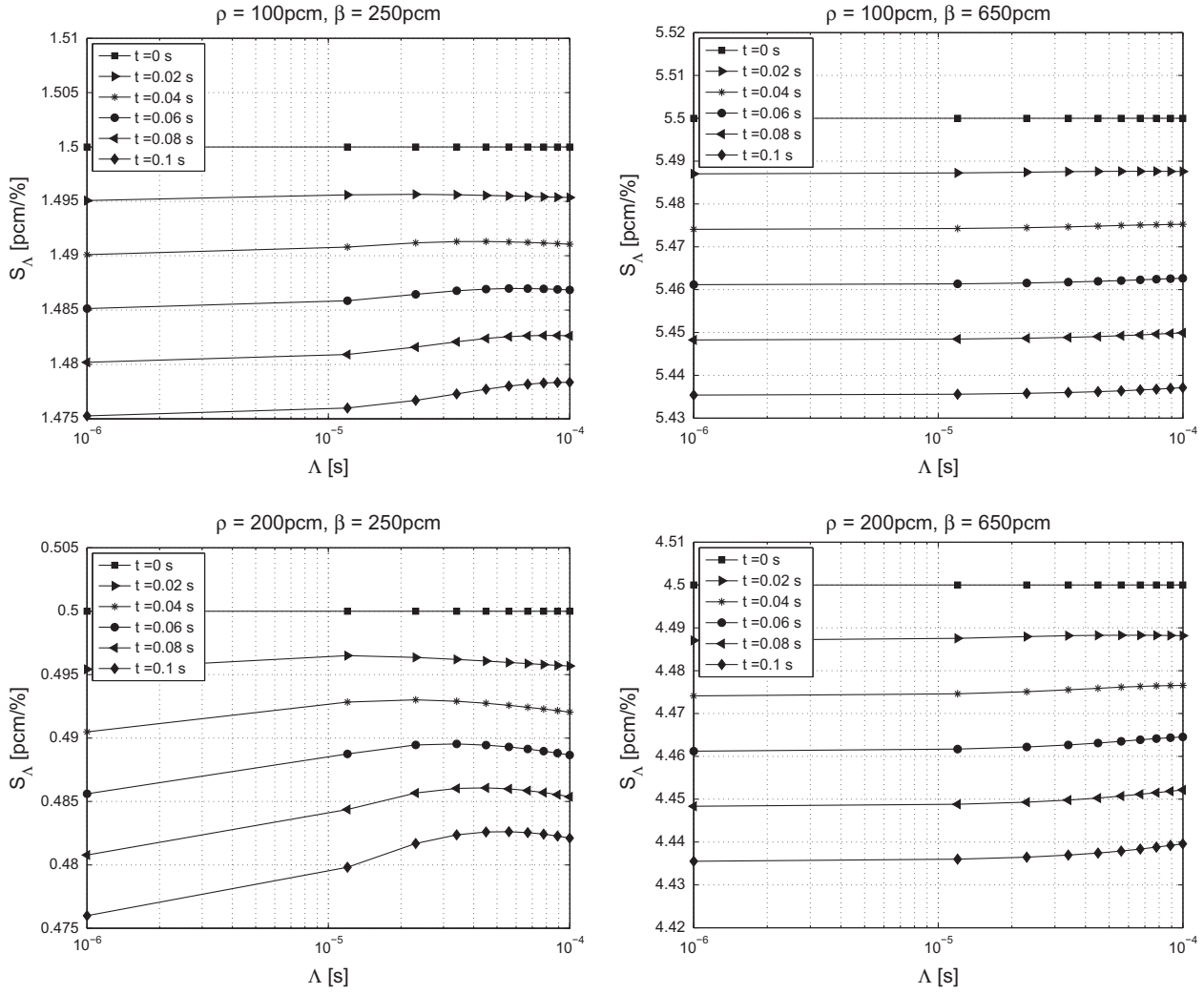

Fig. 2. Sensitivity S_Λ for different values of ρ and β and for various times.

Table 5
Values of β_i and λ_i. The basic nuclear data are retrieved from the JEFF-3.1.1 Nuclear Data Library.

i	λ_i (s^{-1})	β_i (pcm)
1	0.012467	23.33
2	0.028292	101.62
3	0.042524	68.18
4	0.13304	140.79
5	0.29247	230.84
6	0.66649	86.30
7	1.6348	72.49
8	3.5546	27.88

transient data for the power are generated by an eight-family model. The interpretation is then carried out using either a one-family approach, as described above, or the straightforward generalization to a multiple-family case of formula (13) for the fundamental eigenvalue, i.e.

$$\dot{P}(t) - \omega_1 P(t) = -\sum_{i=1}^{M} \frac{\lambda_i}{\omega_1 + \lambda_i} \left[\dot{C}_i(t) - \omega_1 C_i(t)\right], \tag{22}$$

M being the number of delayed families, and the generalization of Eq. (7) for the inhour equation. In this latter case, ω_1 shall be the largest of the roots of a $(M+1)$-degree algebraic equation. In the one family approach, the equivalent delayed neutron data β and λ are computed by the following simple averaging formulae (Keepin, 1965):

$$\beta = \sum_{i=1}^{M} \beta_i, \quad \frac{\beta}{\lambda} = \sum_{i=1}^{M} \frac{\beta_i}{\lambda_i}. \tag{23}$$

Referring to an eight-family case characterized by the data given in Table 5, as retrieved from the JEFF-3.1.1 Nuclear Data Library (Santamarina et al., 2009), some results are presented in Table 6. The results clearly show that, as expected, using a one-family interpretation a satisfactory value for the reactivity is obtained only in the early portion of the transient, while less and less accurate values are obtained for times in the order of the decay time of the delayed neutron precursors. The values estimated with one equivalent family only become totally unreliable for large values of the time.

4. Spatial effects

In this section the study of spatial transients is carried out. The power signal to be interpreted is generated by a diffusion code that numerically solves the time-dependent multigroup diffusion equations (Dulla et al., 2013). From now on, only one family of delayed neutrons is assumed. The effective fraction of delayed neutrons and the effective prompt generation time have been assumed to take the values obtained from the classical formulae of the Henry procedure leading to the point kinetic model (Henry, 1975). In real experiments, such parameters should be retrieved by independent experimental measurements.

Table 6
Results obtained using 8 families of delayed neutrons as compared to results obtained using one equivalent family.

t (s)	$M = 8$ $(\rho_{exp} - \rho)$ (pcm)	$M = 1$ $(\rho_{exp} - \rho)$ (pcm)
$1.0E-05$	$-5.0E+00$	$-5.0E+00$
$1.0E-04$	$-2.7E+00$	$-2.7E+00$
$1.0E-03$	$-1.6E-02$	$5.1E-02$
$8.0E-03$	$-4.3E-06$	$6.3E-01$
$1.0E-02$	$-4.1E-06$	$7.9E-01$
$2.1E-02$	$-2.2E-04$	$1.7E+00$
$4.1E-02$	$-4.9E-04$	$3.2E+00$
$6.1E-02$	$-7.4E-04$	$4.7E+00$
$1.0E-01$	$-1.2E-03$	$7.6E+00$
$1.1E+00$	$-4.6E-03$	$4.9E+01$
$2.0E+00$	$-5.0E-03$	$6.8E+01$

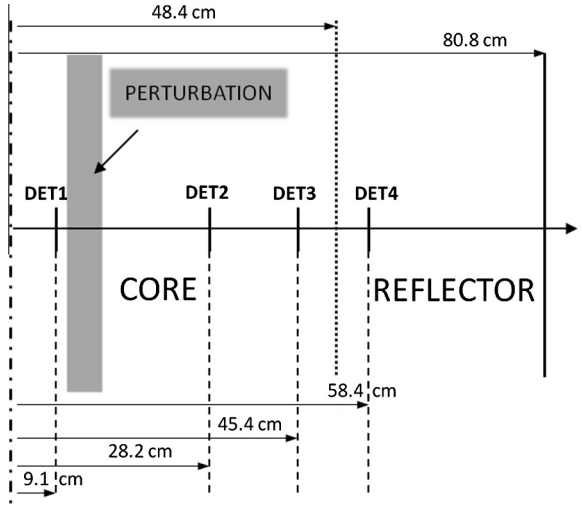

Fig. 3. Scheme of the system studied to analyze the spatial effects. Locations of four flux detectors are indicated (DET1, DET2, DET3 and DET4).

The reflected system considered is shown in Fig. 3, where a localized time-step perturbation is introduced to induce a transient. The nuclear data and the effective integral parameters are given in Table 7, for a one energy group calculation. Four flux detectors are indicated at different positions in the system. From such detectors the flux signal is taken as the amplitude function P to be used for the point kinetic interpretation. A change of the fission cross section of 1% is introduced for the first transient (ρ = 111.6 pcm) and of 3% for the second one (ρ = 35.6 pcm). In Table 8 also the results obtained by using the total power signal are reported, although such information is hardly available in real cases. The analysis of the results for two transients involving different reactivity insertions (Table 8) shows that using the total power signal better results are obtained, as expected. Poor results are obtained for very short times, because the signal is affected by very strong spatial effects. On the other hand, very good results are obtained later in the transients, with slight differences among the various detectors.

It is also worth mentioning that it has been verified that spectral effects are of little importance, since they die out in a very short time following the insertion of a perturbation.

5. Effect of the experimental uncertainty

At last, another important issue concerns the assessment of the robustness of the method against the statistical noise associated to the experimental power signal. The noise may have a deleterious consequence on the evaluation of the signal derivative, which may affect the accuracy of the reactivity prediction. To cope with this problem, existing and well-assessed algorithms to de-noise

Table 7
Nuclear data for the space calculation.

	Core	Reflector
D (cm)	7.6197E-01	1.0721E-01
Σ_a (cm^{-1})	5.8111E-03	1.8976E-04
Σ_f (cm^{-1})	9.5976E-03	0
Λ (s)	9.39E-07	
β (pcm)	751.42	
λ (s^{-1})	0.1	

Table 8
Performance of the method when used for the interpretation of local flux signals during a spatial transient.

ρ (pcm)	t (s)	$(\rho_{exp}-\rho)$ (pcm)				
		P	DET1	DET2	DET3	DET4
112.5	1.00E − 06	1.20E + 01	7.42E + 01	−1.07E + 01	−7.54E + 01	−1.11E + 02
	1.00E − 05	1.19E + 00	3.40E + 00	−5.41E − 01	−4.02E + 00	−1.82E + 01
	1.00E − 04	−2.45E − 01	4.84E − 01	−1.62E + 00	−2.68E + 00	−2.71E + 00
	1.00E − 03	−1.49E − 01	5.26E − 01	−1.42E + 00	−2.40E + 00	−2.42E + 00
	1.00E − 02	−1.42E − 01	5.32E − 01	−1.41E + 00	−2.39E + 00	−2.41E + 00
	1.00E − 01	−1.41E − 01	5.26E − 01	−1.40E + 00	−2.37E + 00	−2.39E + 00
	1.00E + 00	−1.34E − 01	4.69E − 01	−1.27E + 00	−2.14E + 00	−2.16E + 00
	1.50E + 00	−1.30E − 01	4.39E − 01	−1.20E + 00	−2.02E + 00	−2.04E + 00
	2.00E + 00	−1.26E − 01	4.12E − 01	−1.13E + 00	−1.91E + 00	−1.93E + 00
338.3	1.00E − 06	3.71E + 01	2.23E + 02	−3.18E + 01	−2.26E + 02	−.33E + 02
	1.00E − 05	5.08E + 00	1.16E + 01	−3.83E − 02	−1.04E + 01	−5.29E + 01
	1.00E − 04	2.30E − 01	2.06E + 00	−3.26E + 00	−5.97E + 00	−6.05E + 00
	1.00E − 03	−2.92E − 01	1.02E + 00	−2.79E + 00	−4.73E + 00	−4.76E + 00
	1.00E − 02	−2.03E − 01	1.10E + 00	−2.68E + 00	−4.61E + 00	−4.64E + 00
	1.00E − 01	−2.01E − 01	1.08E + 00	−2.64E + 00	−4.53E + 00	−4.57E + 00
	1.00E + 00	−1.77E − 01	9.18E − 01	−2.25E + 00	−3.86E + 00	−3.89E + 00
	1.50E + 00	−1.65E − 01	8.37E − 01	−2.06E + 00	−3.53E + 00	−3.56E + 00
	2.00E + 00	−1.54E − 01	7.62E − 01	−1.88E + 00	−3.23E + 00	−3.25E + 00

the experimental signal must be used, in order to regularize the numerical differentiation process.

The effect of the noise is studied in the following, restricting to a fully point-like system. The values of the power at the detection times along the transient are perturbed statistically according to a uniform distribution around the deterministic value (white noise). The amplitude δ of the interval on which the constant probability density function is assumed to be non-zero is a simulated measure of the dispersion of the experimental data. Using such a probability density function it is possible to sample the values of the power at each detection time, and thus to sample transient histories in a Monte Carlo-like fashion.

To reconstruct the reactivity using the statistically perturbed power values, it is crucial to use an adequate approach to evaluate the derivative of the power. In the present calculations an efficient algorithm to differentiate noisy functions is used (Chartrand, 2011). Once the samples of the power histories are generated, the estimates of the reactivity at each instant can be obtained for each history, from which sample averages and standard deviations can also be easily obtained. This exercise is certainly useful to get some information on how the statistical uncertainty on the power propagates to the estimate of the reactivity. However, it may also be interesting to consider a single power history, to have an idea on what can be expected in a single experimental observation on a specific power evolution.

Some results for the full statistical analysis are presented in Table 9, considering two transients associated to different values of the reactivity. A single history analysis is presented in Table 10.

Table 10
Single history reactivity prediction.

δ (%)	t (s)	$(\rho_{exp}-\rho)$ (pcm)	
		ρ = 50 pcm	ρ = 300 pcm
1	1.0E − 04	−3.00E + 01	−1.63E + 02
	5.0E − 04	−1.76E + 00	−3.22E + 01
	7.5E − 04	−5.41E − 01	−8.84E + 00
	1.0E − 03	−6.45E + 00	−5.45E + 00
	1.0E − 02	−6.19E + 00	1.14E + 00
5	1.0E − 04	−5.39E + 01	−1.72E + 02
	5.0E − 04	1.17E + 01	−9.09E + 00
	7.5E − 04	7.25E + 00	3.50E + 00
	1.0E − 03	−1.97E + 01	−6.99E + 00
	1.0E − 02	−2.40E + 01	1.56E + 01

The differentiation algorithm adopted has proved to yield satisfactory results, except at the very beginning of the transient following the reactivity perturbation.

6. Conclusions and future developments

A new method to monitor the instantaneous reactivity in a source-free nuclear reactor is developed and tested. The method is based on the relationship that can be established between the quantities involving the power signal and the stable period of the

Table 9
Statistical analysis of the effect of the power signal noise. $\langle \rho_{exp} \rangle$ denotes the sample average of the reactivity and σ_ρ the corresponding standard deviation. Ten thousand power histories are considered.

δ (%)	t (s)	ρ = 50 pcm		ρ = 300 pcm	
		$\langle \rho_{exp} \rangle$ (pcm)	σ_ρ (pcm)	$\langle \rho_{exp} \rangle$ (pcm)	σ_ρ (pcm)
1	1.0E − 04	2.35E + 01	5.13E + 00	1.31E + 02	4.22E + 00
	5.0E − 04	4.77E + 01	4.94E + 00	2.69E + 02	3.11E + 00
	7.5E − 04	4.94E + 01	4.89E + 00	2.88E + 02	2.94E + 00
	1.0E − 03	4.98E + 01	4.89E + 00	2.95E + 02	2.89E + 00
	1.0E − 02	5.00E + 01	4.89E + 00	3.00E + 02	2.86E + 00
5	1.0E − 04	2.28E + 01	2.55E + 01	1.31E + 02	2.11E + 01
	5.0E − 04	4.69E + 01	2.46E + 01	2.69E + 02	1.55E + 01
	7.5E − 04	4.86E + 01	2.44E + 01	2.87E + 02	1.47E + 01
	1.0E − 03	4.92E + 01	2.46E + 01	2.95E + 02	1.45E + 01
	1.0E − 02	4.91E + 01	2.46E + 01	2.99E + 02	1.43E + 01

system within the point kinetic model. The other kinetic parameters (effective delayed neutron fraction and prompt generation time) must be available through independent measurements.

The new method is assessed by studying its performance assuming the power signal is available on a given time grid. The sensitivity to the uncertainty of the kinetic parameters is also investigated. Furthermore, the performance when spatial and spectral effects are present is discussed. The method proves to be very accurate and robust in predicting the system reactivity under the various transient conditions that are considered. Furthermore, the effect of the experimental noise on the accuracy of the reactivity prediction is investigated, proving that satisfactory results can be obtained by the use of adequate differentiation algorithms.

The method herewith presented can be extended also for applications to source-driven subcritical systems. Many methods are available for the measurement of the subcriticality level by acting on the source, for instance by performing pulsed or oscillated experiments. However, these techniques are obviously not suitable for continuous monitoring. Therefore, the extension of the present technique looks particularly significant, because the on-line reactivity monitoring is a crucial aspect for the development of accelerator-driven subcritical systems, since it is mandatory for safety considerations to have a reliable information on the subcriticality margin at each instant of the operation of the system. For the extension of the method it is necessary to re-formulate the procedure starting from the source-driven point kinetic equations. For the development of the interpretation procedure it is required to calibrate the intensity of the effective neutron source, referring to the initial, reference system.

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